Before normalization the modes are at
ftp://ftpprd.ncep.noaa.gov/pub/cpc/wd51hd/ake/seasonal/jfm19482003/z1000eotn.gif
ftp://ftpprd.ncep.noaa.gov/pub/cpc/wd51hd/ake/seasonal/jfm19482003/z1000eofn.gif
ftp://ftpprd.ncep.noaa.gov/pub/cpc/wd51hd/ake/seasonal/jfm19482003/z1000eotr.gif
ftp://ftpprd.ncep.noaa.gov/pub/cpc/wd51hd/ake/seasonal/jfm19482003/z1000eofr.gif

After normalization the modes are at
ftp://ftpprd.ncep.noaa.gov/pub/cpc/wd51hd/ake/seasonal/jfm19482003/normed_z1000eotn.gif
ftp://ftpprd.ncep.noaa.gov/pub/cpc/wd51hd/ake/seasonal/jfm19482003/normed_z1000eofn.gif
ftp://ftpprd.ncep.noaa.gov/pub/cpc/wd51hd/ake/seasonal/jfm19482003/normed_z1000eotr.gif
ftp://ftpprd.ncep.noaa.gov/pub/cpc/wd51hd/ake/seasonal/jfm19482003/normed_z1000eofr.gif

About the graphical presentation of EOT and EOF, units, normalization etc. Let's assume that a data set X(s,t) can be represented by

$$X(s,t) = \sum_m \alpha_m(t)\, e_m(s) \qquad (1 - \text{normal})$$

or alternatively by

$$X(s,t) = \sum_m \alpha_m(s)\, e_m(t) \qquad (1 - \text{reverse})$$

where (1 - normal) applies to a 'normal' set-up, and (1 - reverse) to a space time reversed set-up. Expressions (1) are valid for both EOT and EOF. As in Van den Dool et al(2000), the physical units of X are carried by α, while e is a non-dimensional regression coefficient. In general neither e, nor α has unit norm.

We now discuss some (non-essential) postprocessing, which in a nutshell is a matter of finding a factor c by which to divide e and multiply α. The l.h.s. of (1) does not change in this operation:

$$X(s,t) = \sum_m \alpha_m(t) * c_n(m)\;\; e_m(s) \,/\, c_n(m) \qquad (2 - \text{normal})$$

$$X(s,t) = \sum_m \alpha_m(s) * c_r(m)\;\; e_m(t) \,/\, c_r(m) \qquad (2 - \text{reverse})$$

Factor c is a function of m, and, c is different depending on whether we start with normal or reverse set-up, hence the subscript n and r. The reasons for doing these extra manipulations are varied. One could wish to have unit vectors in either space or time. Another reason is to make it more graphically obvious that EOFs obtained by normal and reverse calculation are indeed identically the same, as claimed by Van den Dool et al(2000). At any rate we have decide to do a postprocessing which makes, in all four (EOF/EOT, normal/ reverse) possible cases, the maps of unit norm, and places the variance and physical units in the time series.

Thus note the following:

1) We plot maps and time-series consistent with (1) and (2). We do not plot correlations on the map (as is very customary, and we did before ourselves), because e is really a regression coefficient.

2) To say that a mode m calculated under (1) is the same as mode m calculated under (1a) only means that $\alpha_m(t)\, e_m(s) = \alpha_m(s)\, e_m(t)$; but $e_m(s)$ and $\alpha_m(s)$ are not the same, in general. But with the appropriate c applied $\alpha_m(t)*c_n(m) = e_m(t) / c_r(m)$ and $\alpha_m(s)*c_r(m) = e_m(s) / c_n(m)$

3) In order to force two identical modes to actually look identically the same, we do the following. We divide the map at each point by its spatial norm (and multiply the time series by this same norm so as to maintain the l.h.s. as in (2)). The spatial norm of $e_m(s)$ is defined as

$$c_n(m) = \{ \int e_m(s)\, e_m(s)\, ds \}^{1/2}, \quad \text{similarly}$$

$$c_r(m) = \{ \int \alpha_m(s)\, \alpha_m(s)\, ds \}^{1/2}$$

where the integral is over space. This action would make all maps of unit norm and places the variance in the time series. The plots thus show, for example, $\alpha_m(t)*c_n(m)$ as the time series.

4) One can still tell how the calculation was performed, depending whether base-points or seed years are mentioned in the label.

Footnote. It turned out to be slightly unsatisfactory vis-a-vis the contouring package to plot the normalized maps, so for cosmetics, we divide the map at each point by its absolute maximum value - this procedure creates maps with a maximum value of +/-1 (nearly always +1).

Van den Dool, June 13, 2003